Dynamic Pricing and Stabilization of Supply and Demand in Modern Power Grids

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How should it be done?

Dynamic Pricing

Various forms of Dynamic Pricing:

1. Time of Use Pricing
2. Critical Peak Pricing
3. Real Time Pricing

Borenstein et al [1]:

"We conclude by advocating much wider use of dynamic retail pricing, under which prices faced by end-use customers can be adjusted frequently and on short notice to reflect changes in wholesale prices."

"The goal of the RTP can be to reflect wholesale prices or to transmit even stronger retail price incentives...An RTP price might also differ between locations to reflect local congestion, reliability, or market power factors."

"…Such price-responsive demand holds the key to mitigating price volatility in wholesale electricity spot markets."

1. S. Borenstein, et al. Dynamic pricing, advanced metering, and demand response in electricity markets. Retrieved from: http://escholarship.org/uc/item/11w8d6m4

Dynamic Pricing

Various forms of Demand Response [2]:

- RTP DR   2. Explicit Contract DR   3. Imputed DR

↓

Consumers pay the LMP for their marginal consumption.

W. Hogan [2]:

"…any consumer who is paying the RTP for energy is charged the full LMP for its consumption and avoids paying the full LMP when reducing consumption."

"Expanding the use of dynamic pricing, particularly real-time pricing, to provide smarter prices for the smart grid would be a related priority...."

2. W. Hogan. Demand response pricing in organized wholesale markets. IRC Comments, Demand Response Notice of Proposed Rulemaking, FERC Docket RM10-17-000.

The Independent System Operator (ISO)

- ❑ Non-for-profit organization

- ❑ Operates the wholesale markets and the TX grid

- ❑ Primary function is to optimally match supply and demand -- adjusted for reserve -- subject to network constraints.

- ❑ Operation of the real-time markets involves solving a constrained optimization problem to maximize the aggregate benefits of the consumers and producers. (The Economic Dispatch Problem (EDP))

- ❑ In real-time, the objective usually is to minimize total cost of dispatch for a fixed demand

- ❑ Constraints are : KVL, KCL, TX line capacity, generation capacity, local and system-wide reserve, other ISO-specific constraints.

Power Systems

The Independent System Operator (ISO)

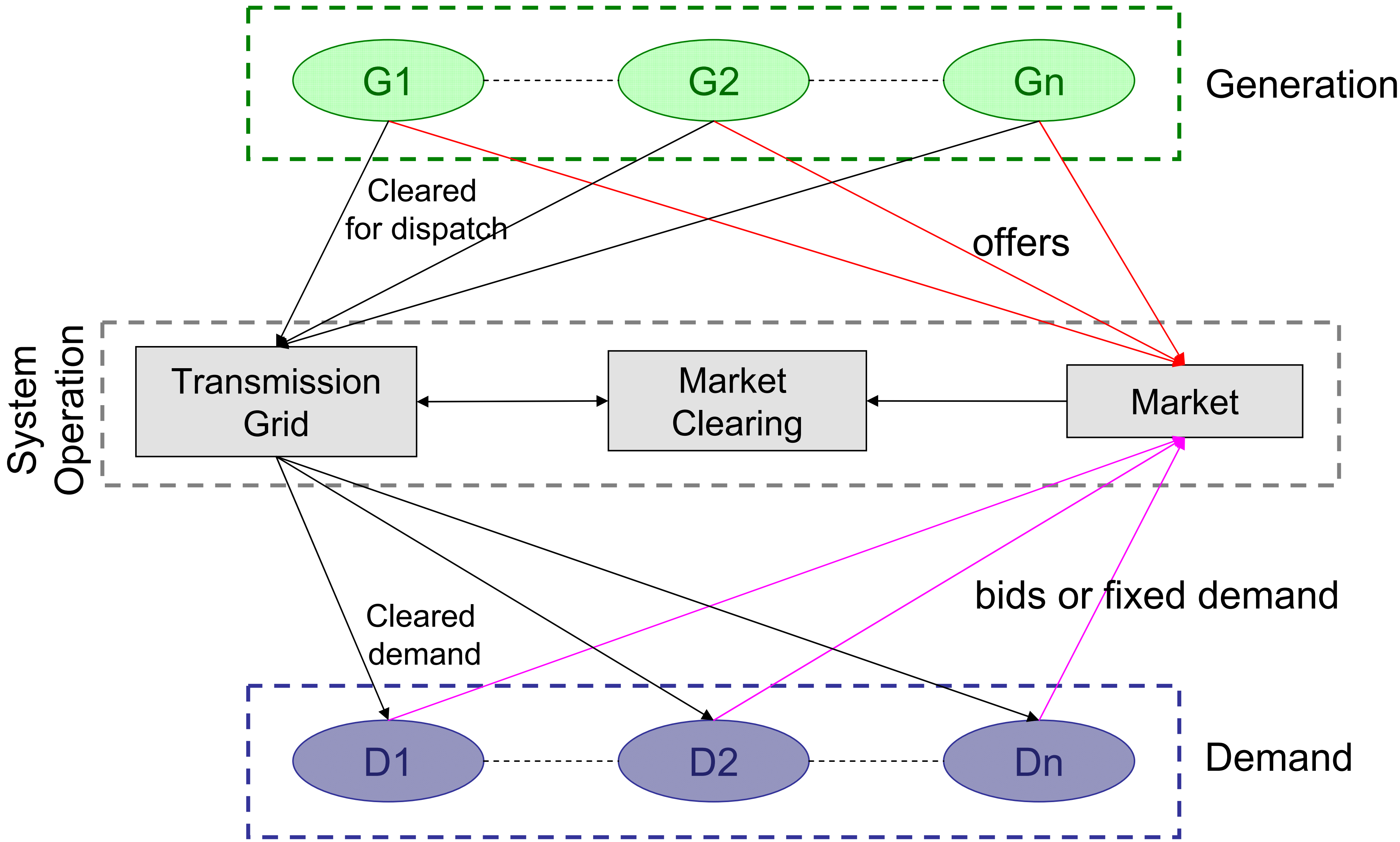
California ISO Control Room in Folsom – photo by *Donald Satterlee*

Courtesy of the California ISO: http://www.caiso.com

CA ISO operates 25,000 circuit miles of high-voltage, long distance power lines

Power Systems

Wholesale Markets and System Operation



- Simplified DC Model:

Generators as ideal current sources

Demand is fixed

$$\min \quad \sum_{i=1}^{n} c_i S_i$$

$$\text{s.t.} \quad \begin{bmatrix} K & E \\ 0 & R \end{bmatrix} \begin{bmatrix} S \\ I \end{bmatrix} = \begin{bmatrix} D \\ 0 \end{bmatrix}$$

KCL

KVL

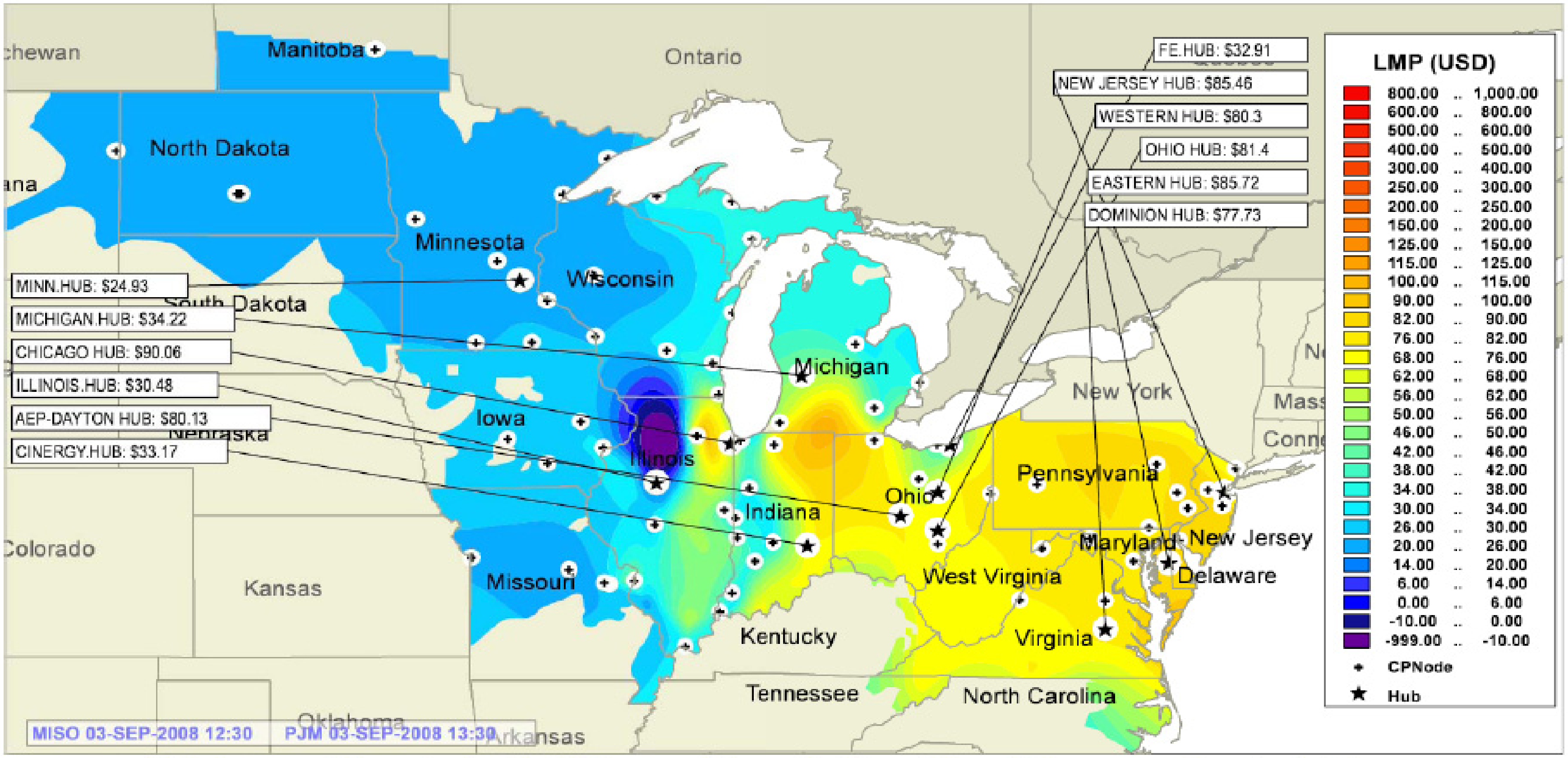
Line Capacity limits $\longrightarrow$ $-I_{\max} \leq I \leq I_{\max}$

Generation limits $\longrightarrow$ $S_{\min} \leq S \leq S_{\max}$

- Locational Marginal Prices are the dual variables corresponding to the constraint $KS + EI = D$.

Locational Marginal Prices

PJM ISO 03-SEP-2008 13:30



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Midwest ISO Market data is based on Eastern Standard Time (EST) while PJM Market data is based on Eastern Prevailing Time.

Midwest ISO We manage power. pjm

Economic Dispatch

Primal and dual problems

dual

Dual objective $\mathcal{D}(\lambda)$ depends on d : $g(\lambda, d)$

$$\mathcal{D}(\lambda) = \boxed{\min_{S,I} \sum_{i=1}^{n} c_i(S_i) - \lambda(KS + EI - d)}$$

$$\text{s.t.} \quad RI = 0$$

$$-I_{\max} \leq I \leq I_{\max}$$

$$S_{\min} \leq S \leq S_{\max}$$

primal

Primal objective: $f(S)$

$$\min_{S,I} \boxed{\sum_{i=1}^{n} c_i(S_i)}$$

$$\text{s.t.} \quad \begin{bmatrix} K & E \\ 0 & R \end{bmatrix} \begin{bmatrix} S \\ I \end{bmatrix} = \begin{bmatrix} d \\ 0 \end{bmatrix}$$

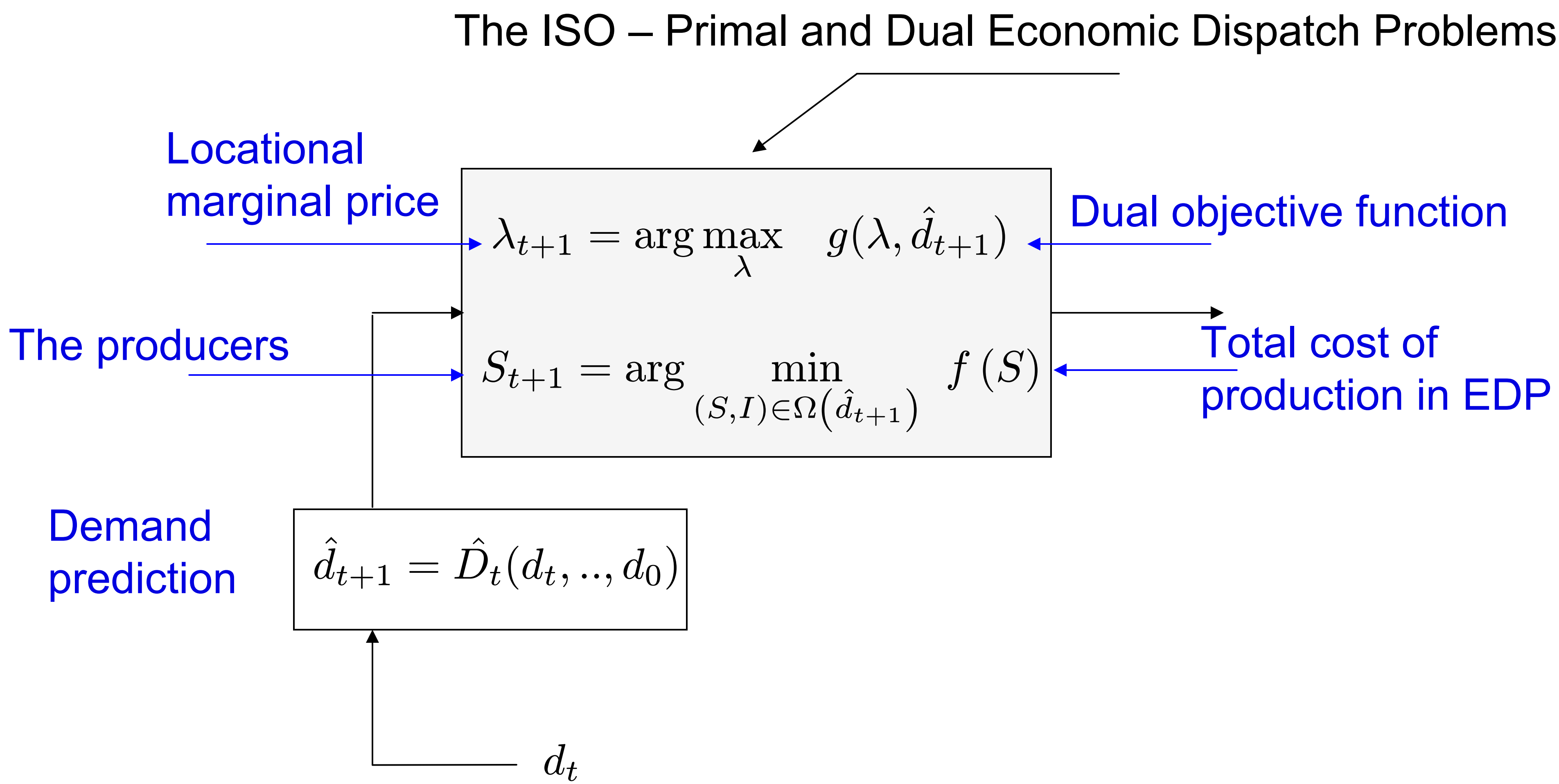
$$-I_{\max} \leq I \leq I_{\max}$$

$$S_{\min} \leq S \leq S_{\max}$$

Primal feasible set: $\Omega(d)$

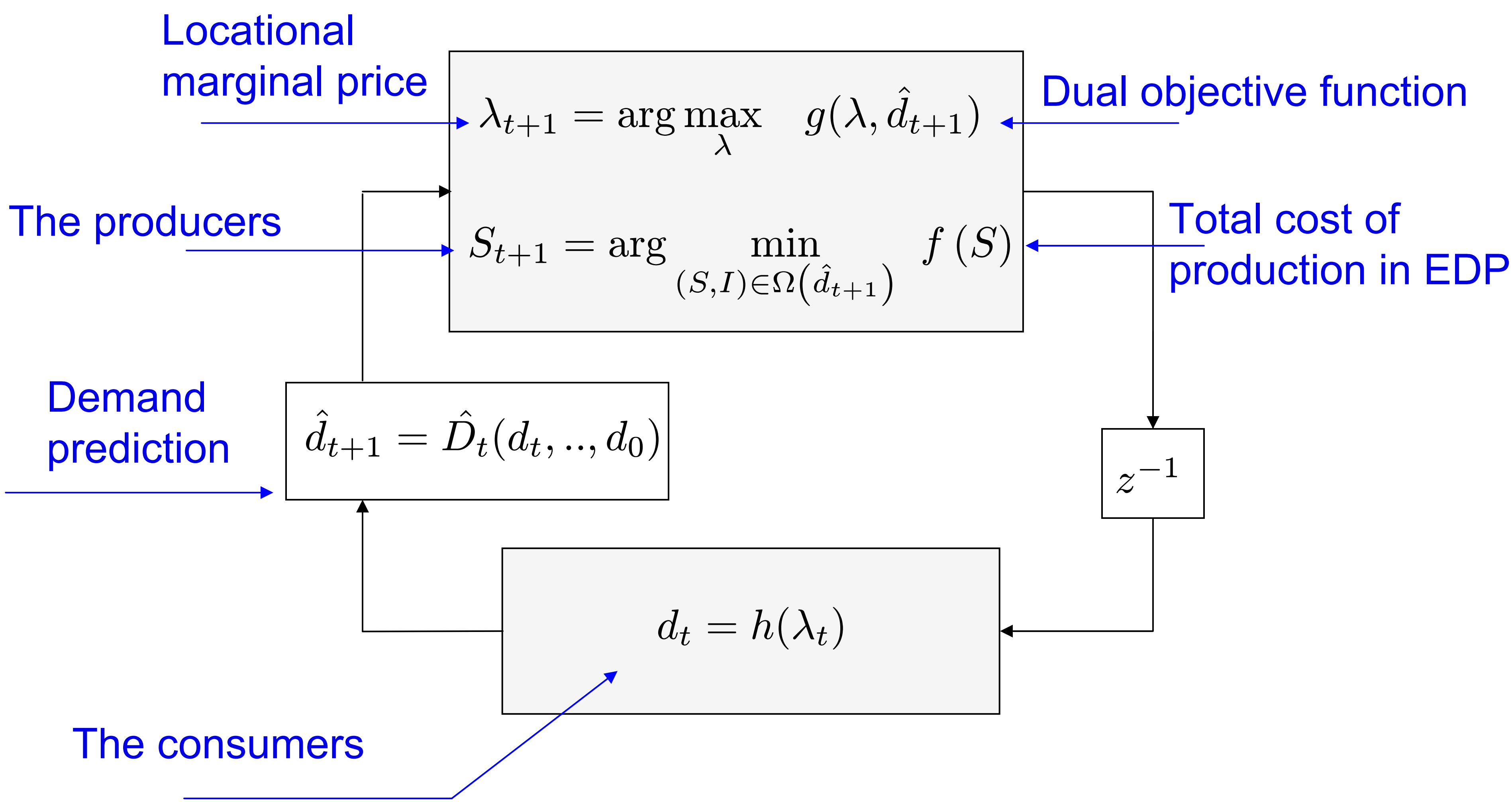
Passive Consumption

The System is Open Loop



Real-Time Pricing

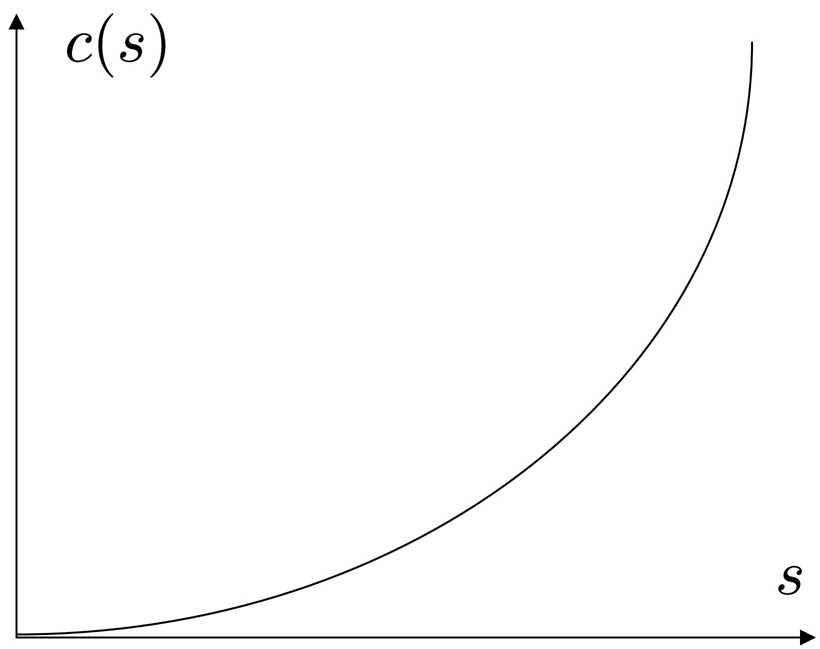
Closing the Loop



Consumers and Producers

Cost functions and value function

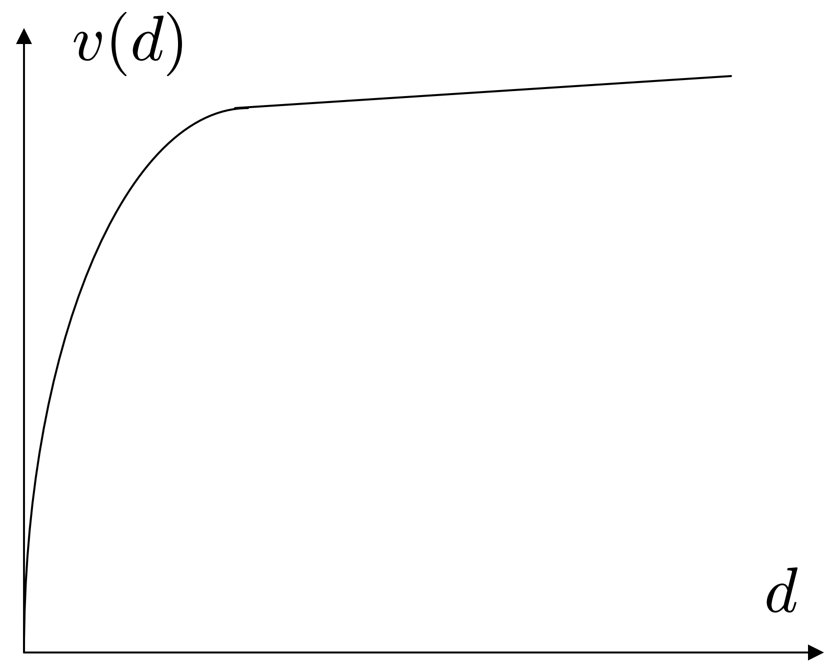
producers



Convex cost function

$$s_t = \arg\max_x \; \lambda_t x - c(x)$$
$$= \dot{c}^{-1}(\lambda_t)$$

consumers

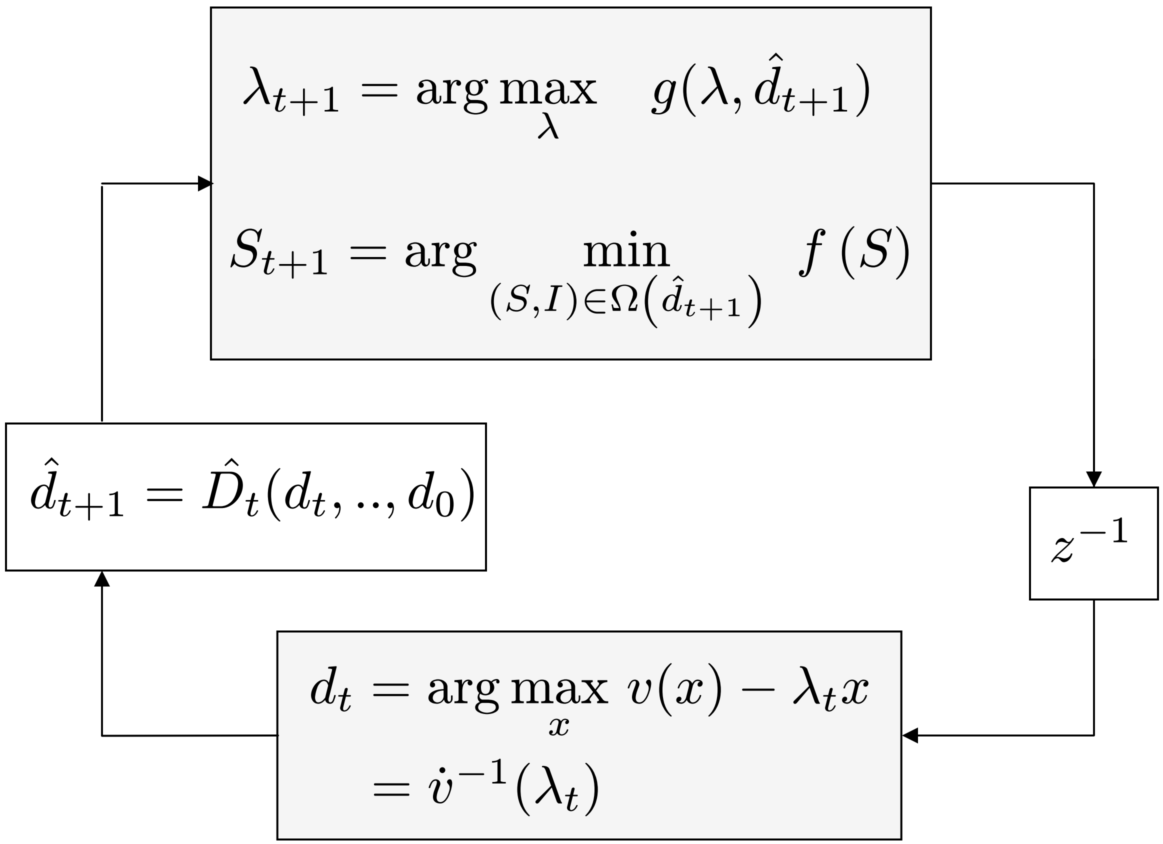


Concave value function

$$d_t = \arg\max_x \; v(x) - \lambda_t x$$
$$= \dot{v}^{-1}(\lambda_t)$$

Real-Time Pricing

Closing the Loop



Dynamic Pricing

Message:

Real time pricing creates a closed loop feedback system

Need good engineering to create a *well-behaved* closed loop system

There are tradeoffs in stability/volatility and efficiency

Closed Loop System Dynamics

Simplified Model

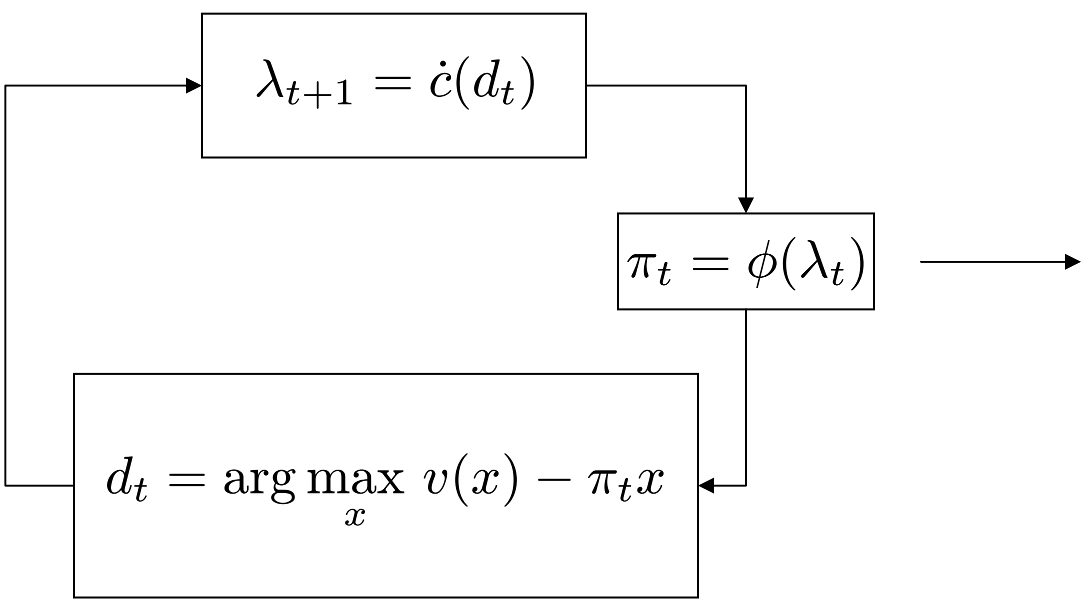
Assumptions:

1. Line capacities are high enough, i.e., no congestion
2. Generator capacities are high enough, i.e., no capacity constraint

Then

1. All the generators can be lumped into one representative generator
2. All the consumer can be lumped into one representative consumer

Assume the Rep. agents' cost (value) functions are smooth convex (concave).



$$\lambda_{t+1} = \dot{c}\left(\dot{v}^{-1}\left(\lambda_t\right)\right)$$

$$\lambda_{t+1} = \dot{c}\left(\dot{v}^{-1}(\phi(\lambda_t))\right)$$

Theorem: The system $x_{k+1} = \psi(x_k)$, where $\psi : \mathbb{R}_+ \to \mathbb{R}_+$, is stable if

there exist functions f and g mapping $\mathbb{R}_+$ to $\mathbb{R}_+$, and $\theta \in (-1, 1)$ satisfying:

$$g(x_{k+1}) = f(x_k) \tag{1}$$

and

$$\left|\dot{f}(x)\right| \leq \theta \dot{g}(x) \tag{2}$$

decomposition of the dynamics

Note: $\boxed{\psi = g^{-1} \circ f}$

In our context: $\psi = \dot{c} \circ \dot{v}^{-1}$, hence, a sufficient stability criterion is:

$$\left|\frac{d}{dx}\dot{v}^{-1}(x)\right| \leq \theta \frac{d}{dx}\dot{c}^{-1}(x)$$

Real-Time Pricing

Stability Criterion

Stability Theorem: The system $\lambda_{t+1} = \dot{c}\left(\dot{v}^{-1}\left(\lambda_t\right)\right)$ is stable if there exists a function $\rho : \mathbb{R}_+ \mapsto \mathbb{R}_+$, and a constant $\theta \in (-1, 1)$, s.t.

$$\left|\frac{\dot{\rho}\left(\dot{v}^{-1}\left(\lambda\right)\right)}{\ddot{v}\left(\dot{v}^{-1}\left(\lambda\right)\right)}\right| \leq \theta \frac{\dot{\rho}\left(\dot{c}^{-1}\left(\lambda\right)\right)}{\ddot{c}\left(\dot{c}^{-1}\left(\lambda\right)\right)} \quad , \quad \forall \lambda \in \mathbb{R}_+ \qquad (1)$$

In particular,

$$\left|\ddot{c}\right| \leq \theta \ddot{v}$$

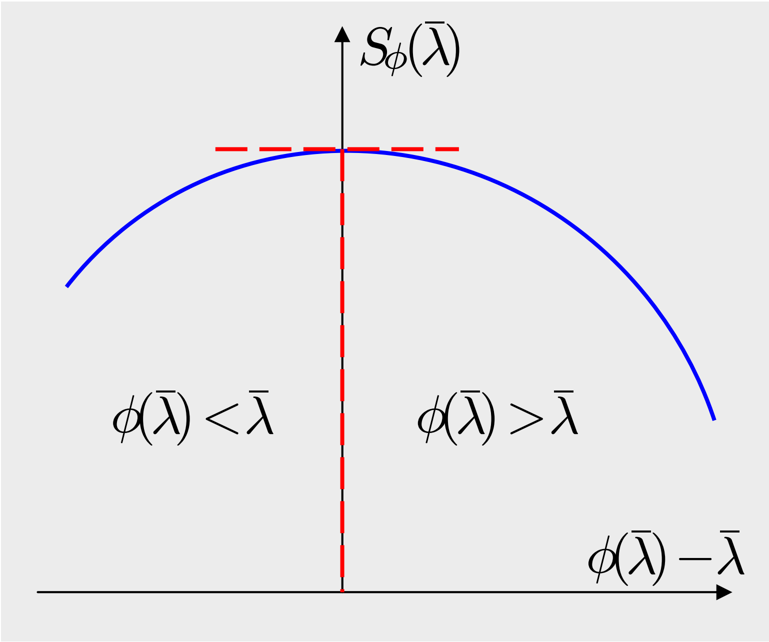
is sufficient. (obtained with $\rho = \dot{c}$)

Real-Time Pricing with a Static Pricing Function

Efficiency Loss

The function $\phi : \mathbb{R}_+ \mapsto \mathbb{R}_+$ stabilizes the system $\lambda_{t+1} = \dot{c}\left(\dot{v}^{-1}\left(\phi(\lambda_t)\right)\right)$ if

$$|\dot{\phi}(\lambda)| \left| \frac{\dot{\rho}\left(\dot{v}^{-1}\left(\phi(\lambda)\right)\right)}{\ddot{v}\left(\dot{v}^{-1}\left(\phi(\lambda)\right)\right)} \right| \leq \theta \frac{\dot{\rho}\left(\dot{c}^{-1}\left(\phi(\lambda)\right)\right)}{\ddot{c}\left(\dot{c}^{-1}\left(\phi(\lambda)\right)\right)}, \qquad \forall \lambda \in \mathbb{R}_+ \tag{2}$$



$$S(x) = v(x) + c(x)$$

$$S_\phi(\lambda) = c(\dot{c}^{-1}(\lambda)) + v(\dot{v}^{-1}(\phi(\lambda)))$$

The farther the wholesale and retail prices at the equilibrium, the more is the efficiency loss.

Real-Time Pricing, A Dynamic Strategy

Stable for sufficiently small γ:

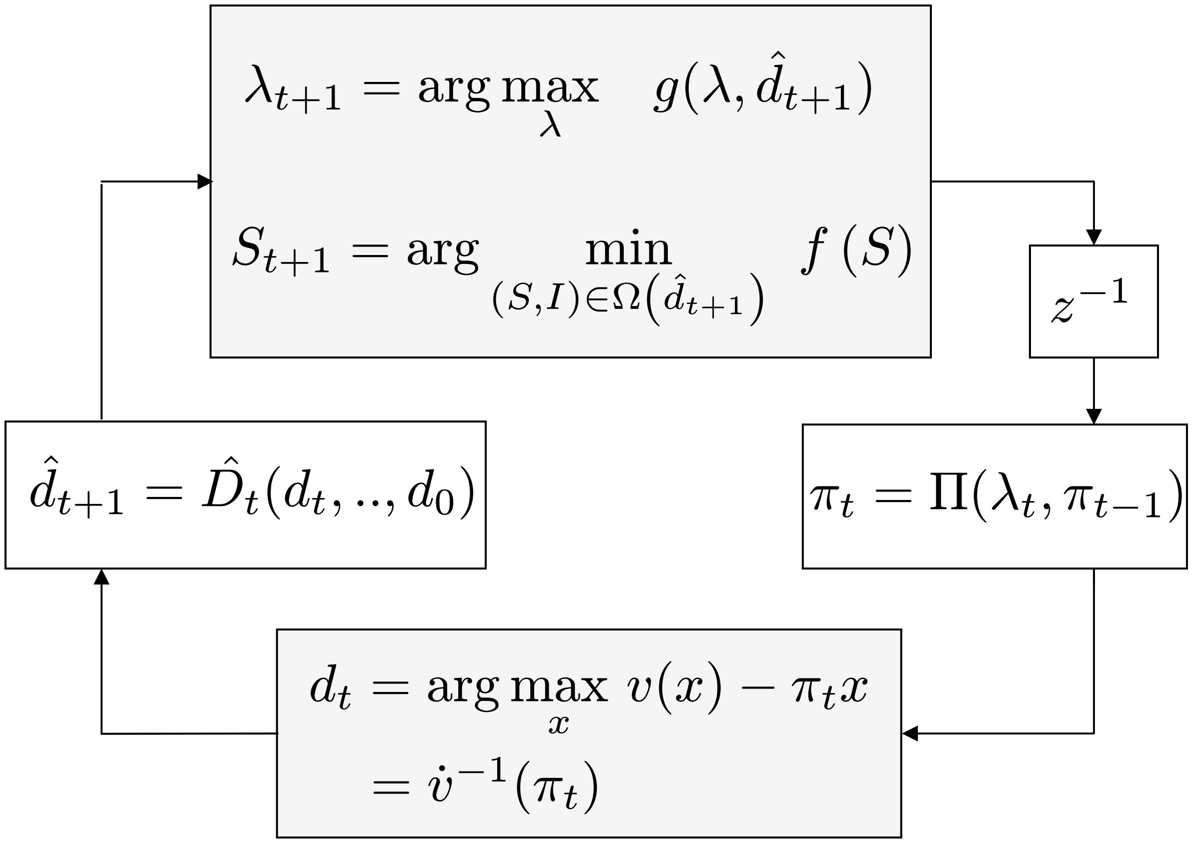
$$\lambda_{t+1} = \lambda_t + \gamma(\dot{c}\left(\dot{v}^{-1}\left(\lambda_t\right)\right) - \lambda_t)$$

Retrieve the original dynamics when $\gamma = 1$

The idea can be used to construct a stabilizing sub-gradient algorithm for the full model of EDP with DC OPF constraints

Real-Time Pricing

Subgradient-based Stabilizing Pricing Mechanism



dual subgradient direction

$$\mathcal{G}(\pi_t) = -Ks_t - EI_t + d_t$$

$$= -Ks_t - EI_t + \dot{v}^{-1}(\pi_t)$$

Real-Time Pricing

Subgradient-based Stabilizing Pricing Mechanism

dual

$$\mathcal{D}(\lambda) = \min_{s,I} \sum_{i=1}^{n} c_i (S_i) - \lambda (KS + EI - d)$$

$$\text{s.t.} \quad RI = 0$$

$$-I_{\max} \leq I \leq I_{\max}$$

$$S_{\min} \leq S \leq S_{\max}$$

primal

$$\min \sum_{i=1}^{n} c_i(S_i)$$

$$\text{s.t.} \quad \begin{bmatrix} K & E \\ 0 & R \end{bmatrix} \begin{bmatrix} S \\ I \end{bmatrix} = \begin{bmatrix} d \\ 0 \end{bmatrix}$$

$$-I_{\max} \leq I \leq I_{\max}$$

$$S_{\min} \leq S \leq S_{\max}$$

The dual is concave and non-differentiable

$-Ks_t - EI_t + d_t$ is a subgradient direction

Real-Time Pricing

Subgradient-based Stabilizing Pricing Mechanism

$$\begin{aligned}
\mathcal{D}(\lambda) = \quad & \min_{s,I} \quad \sum_{i=1}^{n} c_i(S_i) - \lambda(KS + EI - d) \\
& \text{s.t.} \quad RI = 0 \\
& \qquad -I_{\max} \le I \le I_{\max} \\
& \qquad S_{\min} \le S \le S_{\max}
\end{aligned}$$

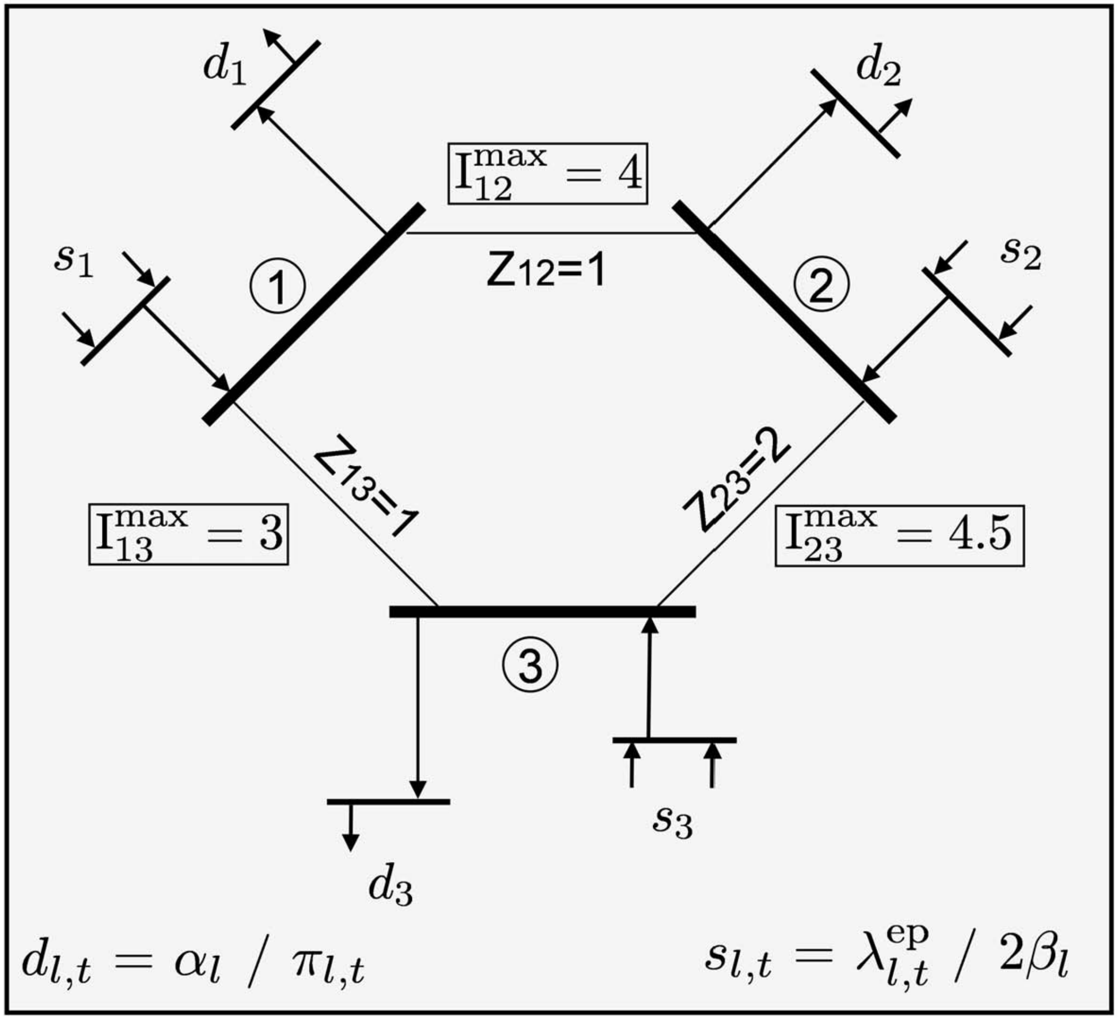
Theorem: The pricing mechanism

$$\begin{aligned}
\pi_{t+1} &= \pi_t + \gamma G(\pi_t) \\
&= \pi_t - \gamma(KS_t + EI_t - d_t)
\end{aligned}$$

stabilizes the system: For sufficiently small γ, (π_t, S_t, d_t) converge to a small neighborhood of (π^*, S^*, d^*) where π^* is the dual optimal solution and S^* and d^* are the corresponding optimal supply and demand.

Real-Time Pricing

Numerical Simulation



consumer value functions are logarithmic:

$$v_l(d) = \log(d)$$

$$d_l = \alpha_l / \pi_l$$

producer cost functions are quadratic:

$$c_l(s) = \beta_l s^2$$

$$s_l = \lambda_l / (2\beta_l)$$

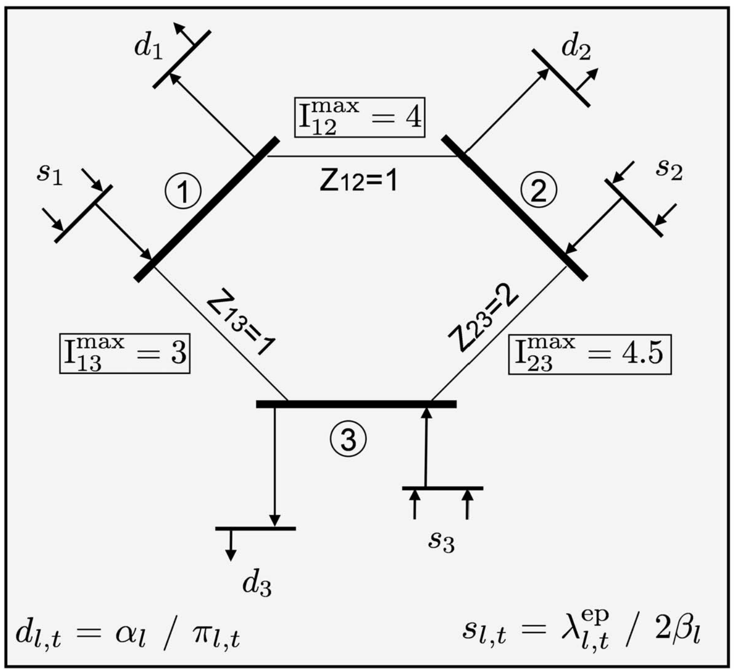
To make the simulations more realistic, we approximated the quadratic costs with piecewise linear functions to get an LP for EDP. Also added noise to α, β

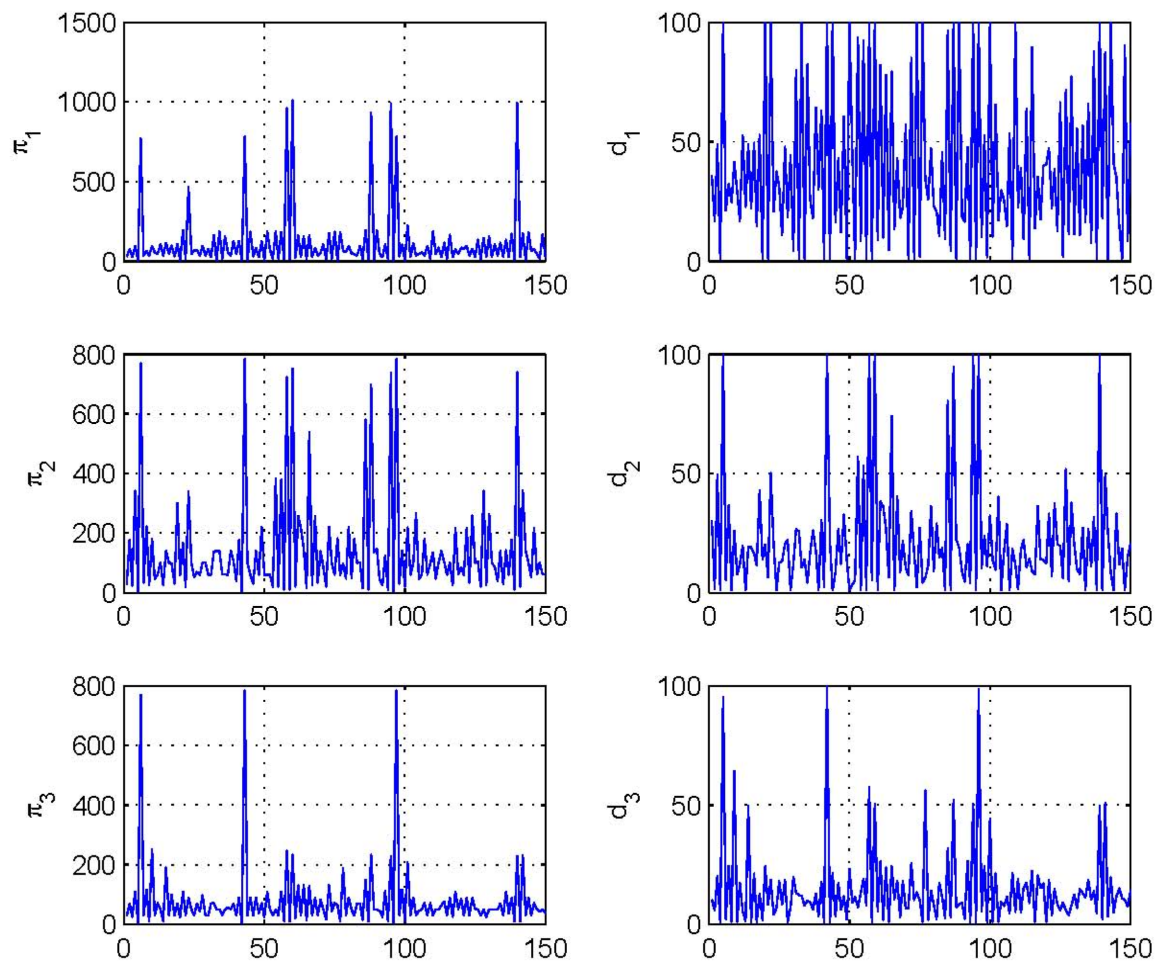
Real-Time Pricing

System Instability

Both demand and price are very volatile

$$\pi_t = \lambda_t$$





3 Bus system

LMP

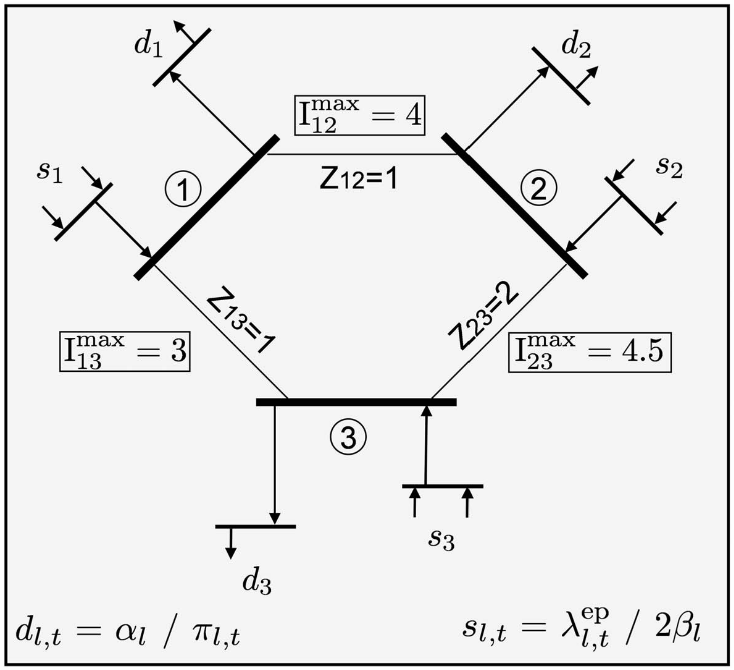
demand

Real-Time Pricing

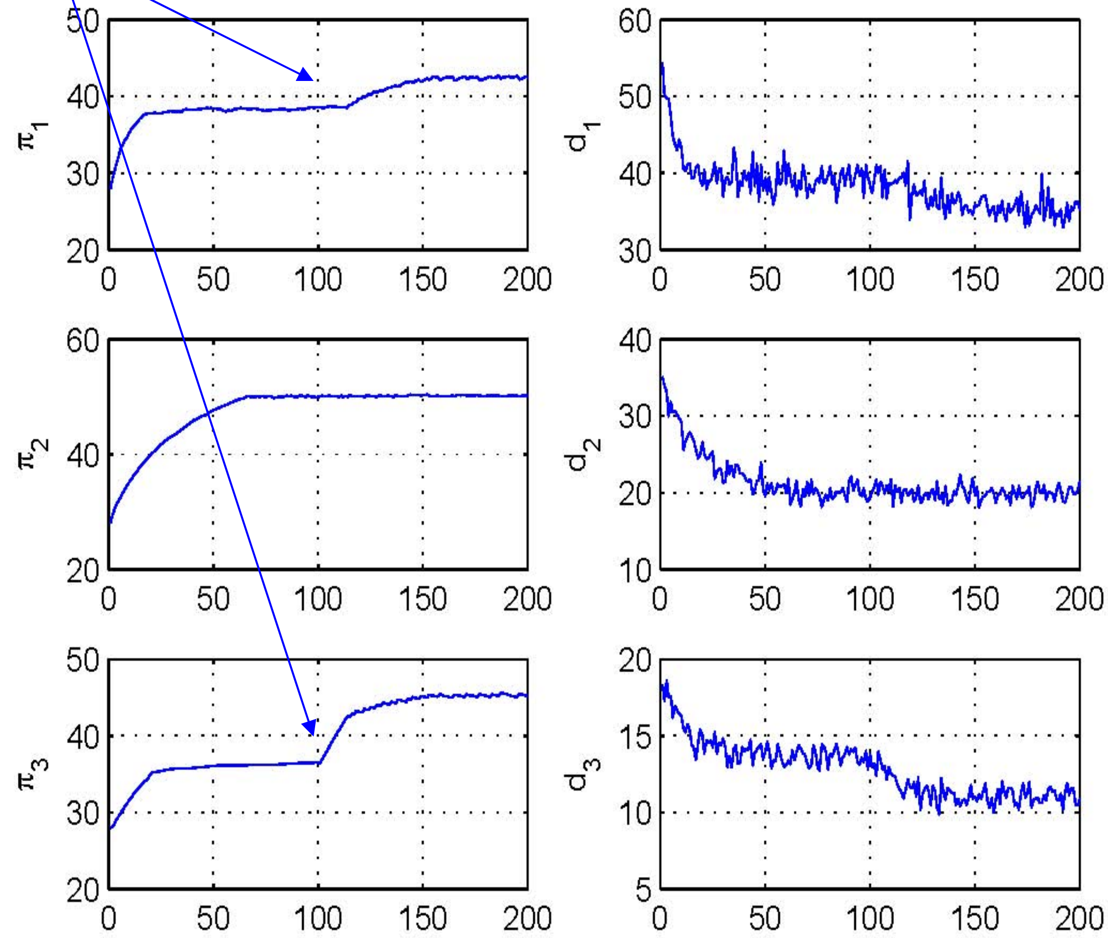
Subgradient-based Stabilizing Pricing Mechanism

disturbance was introduced at node 3 at time t=100

$$\pi_{t+1} = \pi_t + \gamma \mathcal{G}(\pi_t)$$



3 Bus system



LMP demand

Future Work

1. More sophisticated models (consumer behavior, power flow, market clearing)
2. Price anticipating consumers / consumers with rational expectations
3. Incorporate reserve capacity markets in the model
4. Stochastic model of supply / demand
5. Dynamic model of the Economic Dispatch over a rolling time horizon
6. Partial knowledge of demand value function -- demand prediction
7. Tradeoffs between wholesale market volatility and retail price volatility
8. Control always has a cost, in this case real money. There will be discrepancies between retail revenue and wholesale cost. Who pays for it? Consumers? How does that change consumer behavior?
9. Fairness: If only a portion of population is participating in RTP, those with fixed-price contract can drive the prices very high for the RTP consumers at a time of shortage, exposing them to undue risk and inconvenience…

Thank you!